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# 1.1 Introduction to AES

- ▶ Ref: “The Design of Rijndael” (Joan Daemen and Vincent Rijmen) – Springer 2002
- ▶ Rijndael announced as the winner of the AES competition in October 2000
  - Designed by Rijmen and Daemen
- ▶ AES is a symmetric cipher
  - Blocksize is 128 bits viewed as a 4 by 4 matrix of bytes
  - Keys are 128, 192 or 256 bits
- ▶ In this talk we concentrate on AES-128 with 128 bit key
- ▶ AES-128 has 10 rounds
- ▶ AES-192 has 12 rounds
- ▶ AES-256 has 14 rounds
- ▶ The 128, 192 or 256 bit key is expanded to provide enough key bits to encrypt each round

## 1.2 AES-128 Round Structure

- ▶ AES-128 works on a State that is 4 by 4 matrix of 8 bit bytes
- ▶ AES(State, CipherKey)

```
{  
  
  KeyExpansion(CipherKey, ExpandedKey)  
  AddRoundKey(State, ExpandedKey[0])  
  for i=1:9  
    Round(State,ExpandedKey[i])  
  end  
  FinalRound(State, ExpandedKey[10])  
}
```

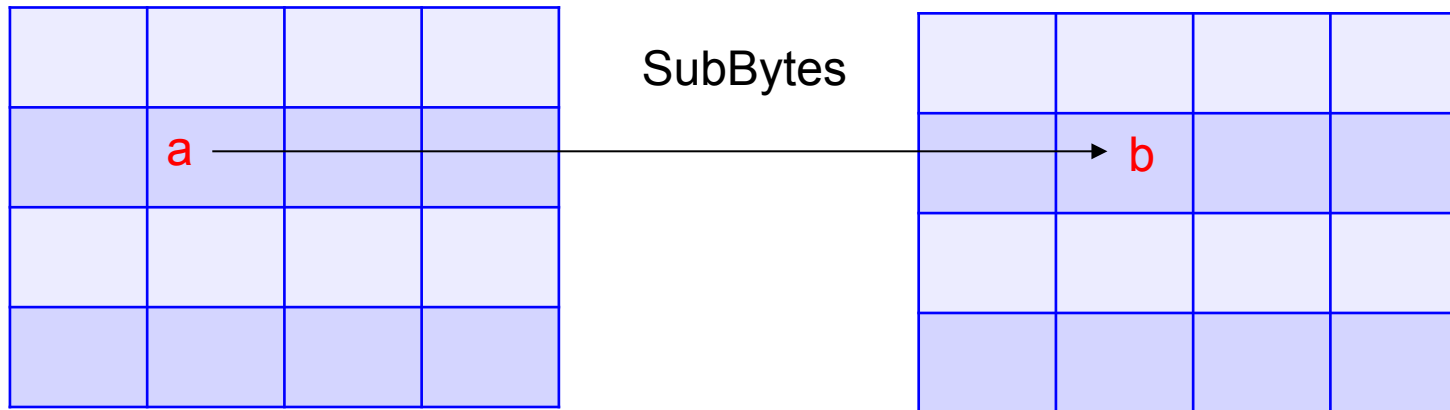
## 1.3 AES-128 Round Structure



```
Round(State, ExpandedKey[i])  
{  
  Subbytes(State)  
  ShiftRows(State)  
  MixColumns(State)  
  AddRoundKey(State, ExpandedKey[i])  
}
```

```
FinalRound(State, ExpandedKey[i])  
{  
  SubBytes(State)  
  ShiftRows(State)  
  AddRoundKey(State, ExpandedKey[i])  
}
```

## 1.4 AES-128 Round Structure - SubBytes



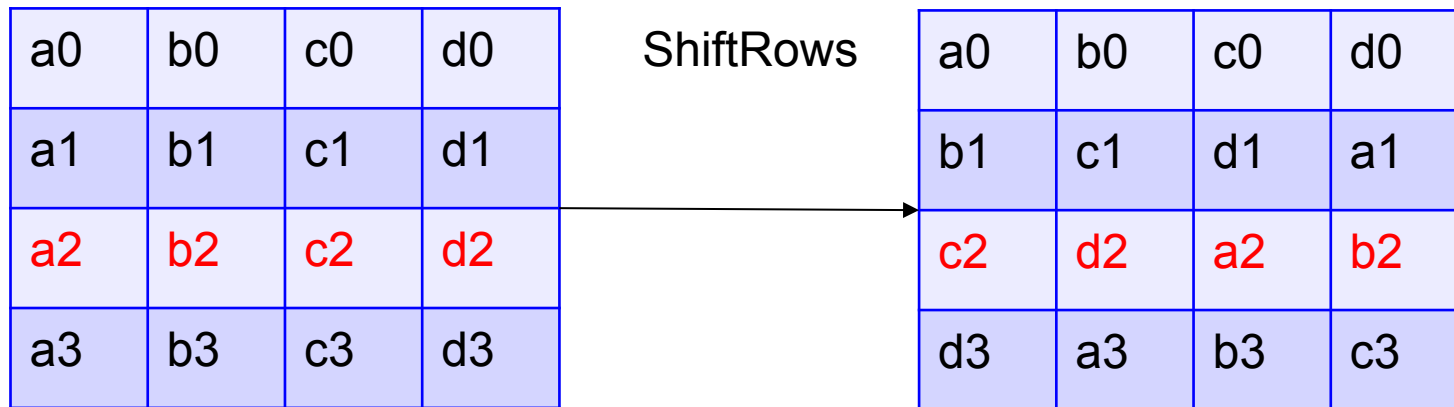
- ▶ SubBytes:  $a \rightarrow f(a^{\uparrow-1}) = b$   
find inverse of  $a$  in  $GF(2^8)$  followed by an affine transformation  $f$   
An affine transformation is a linear mixing and shift of the bits of  $a^{\uparrow-1}$
- ▶ Note 1: SubBytes is the only non-linear part of AES and operates on each byte individually
- ▶ Note 2: XOR is a linear function in  $GF(2)$  and  $GF(2^8)$  which is defined by the irreducible polynomial  $m(x) = x^8 + x^4 + x^3 + x + 1$

## 1.5 AES-128 Round Structure – SubBytes (Contd)

- ▶ SubBytes Affine Transformation

- ▶ 
$$\begin{bmatrix} a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ a_3 & a_2 & a_1 & a_0 & a_7 & a_6 & a_5 & a_4 \\ a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ a_3 & a_2 & a_1 & a_0 & a_7 & a_6 & a_5 & a_4 \\ a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ a_3 & a_2 & a_1 & a_0 & a_7 & a_6 & a_5 & a_4 \\ a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ a_3 & a_2 & a_1 & a_0 & a_7 & a_6 & a_5 & a_4 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} b_7 & b_6 & b_5 & b_4 & b_3 & b_2 & b_1 & b_0 \\ b_3 & b_2 & b_1 & b_0 & b_7 & b_6 & b_5 & b_4 \\ b_7 & b_6 & b_5 & b_4 & b_3 & b_2 & b_1 & b_0 \\ b_3 & b_2 & b_1 & b_0 & b_7 & b_6 & b_5 & b_4 \\ b_7 & b_6 & b_5 & b_4 & b_3 & b_2 & b_1 & b_0 \\ b_3 & b_2 & b_1 & b_0 & b_7 & b_6 & b_5 & b_4 \\ b_7 & b_6 & b_5 & b_4 & b_3 & b_2 & b_1 & b_0 \\ b_3 & b_2 & b_1 & b_0 & b_7 & b_6 & b_5 & b_4 \end{bmatrix}$$

## 1.6 AES-128 Round Structure - ShiftRows



Row 0 is not shifted

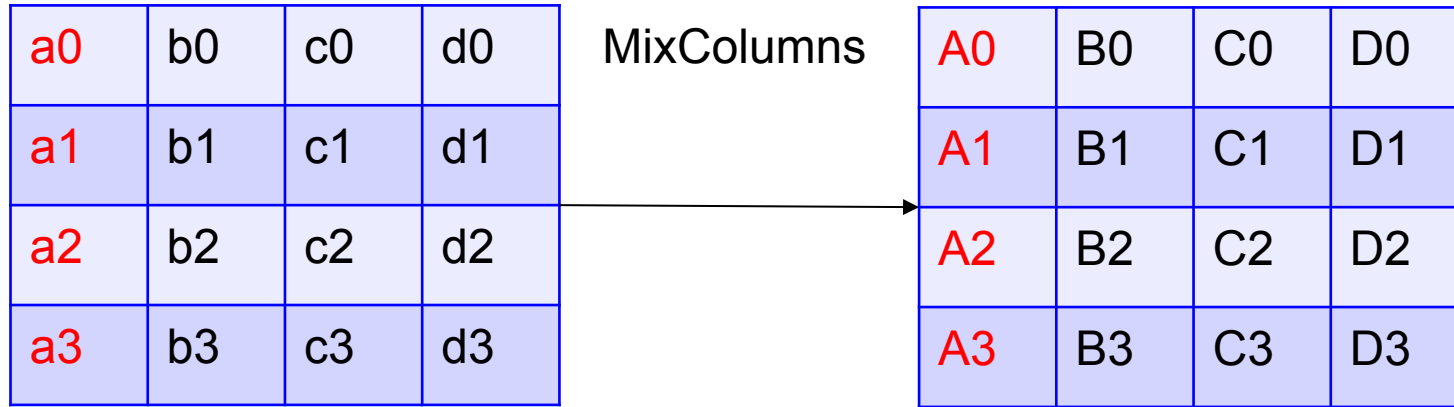
Row 1 is circularly left shifted by 1

Row 2 is circularly left shifted by 2

Row 3 is circularly left shifted by 3

Note: ShiftRows preserves the XOR of the row bytes and the total XOR of all the bytes

## 1.7 AES-128 Round Structure - MixColumns



$$\begin{pmatrix} 2 & 3 & 1 & 2 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 3 & 1 \\ 2 & 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ a_0 & a_1 & a_2 & a_3 \\ a_0 & a_1 & a_2 & a_3 \\ a_0 & a_1 & a_2 & a_3 \end{pmatrix} = \begin{pmatrix} A_0 & A_1 & A_2 & A_3 \\ A_1 & A_2 & A_3 & A_0 \\ A_2 & A_3 & A_0 & A_1 \\ A_3 & A_0 & A_1 & A_2 \end{pmatrix}$$

Addition is performed over  $GF(2)$

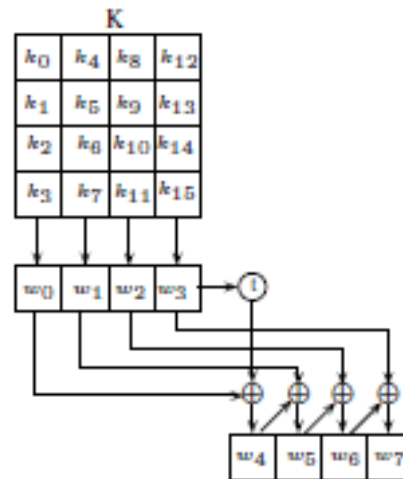
Multiplication over  $GF(2^8)$

Note: MixColumns preserves the XOR of the columns and the total XOR of all the bytes



## 1.8 Key Expansion

- ▶ Ref: “Analysis of Block Cipher Constructions against Biclique and Multiset Attacks”, PhD Thesis by Mohona Ghosh, 2016
- ▶ Key Expansion of AES-128 (example)
- ▶ Begin with 128 bit key prepared as 4x4 byte state array
- ▶ Form 4 32-bit words from the columns. Iterate key schedule to obtain enough key for each of the rounds



## 2.1 Design Principles of AES – Linear Cryptanalysis

- ▶ **Linear Cryptanalysis: Correlations and Linear Trails**

- ▶ The correlation between two Boolean functions  $f$  and  $g$  is

- ▶  $C(f,g) = 2 \cdot \text{Prob}(f(a) = g(a)) - 1$

- ▶  $-1 \leq C(f,g) \leq 1$

- ▶ A parity function is the XOR of a number of bits

- ▶ Given a non-linear function  $S$  we can calculate the correlation of a parity function to it

- ▶ `SubBytes()` has been chosen such that the maximum correlation of a parity function is  $2^{-3}$

- ▶ Given a sequence of rounds we can identify a corresponding linear trail through it which is a sequence of parity functions. We multiply the corresponding correlations of the parity functions to get the correlation of the linear trail. For AES, this gives the maximum of  $2^{-75}$  for the correlation for any four round linear trail.

## 2.2 Design Principles of AES – Differential Cryptanalysis

- ▶ **Differential Cryptanalysis:** Consider two  $n$  bit vectors  $a$  and  $a \oplus \Delta$  where  $a + a \oplus \Delta = \Delta$  a fixed difference pattern.

- ▶ Let  $b = h(a)$ ,  $b \oplus \Delta = h(a \oplus \Delta)$  and  $b + b \oplus \Delta = \Delta$  the difference  $\Delta$  propagates to the difference  $\Delta$ .

- ▶ The difference propagation probability is

$$Prob(\Delta, \Delta) = 2^{-n} \sum_{a \oplus \Delta} \delta(b \oplus \Delta + h(a \oplus \Delta) + h(a))$$

- ▶ The weight of a difference propagation is  $w(\Delta, \Delta) = -\log_2(Prob(\Delta, \Delta))$

- ▶ A differential trail is a sequence of difference patterns:

$$D = (\Delta_0, \Delta_1, \Delta_2, \dots, \Delta_{r-1}, \Delta_r)$$

The weight of a differential trail is the sum of the weights its differential steps

$$w(D) = w(\Delta_0, \Delta_1) + w(\Delta_1, \Delta_2) + \dots + w(\Delta_{r-1}, \Delta_r)$$

SubBytes has a differential weight of at least 6 meaning a differential propagation probability of at least  $2^{-6}$ . This gives a minimum weight of 150 for any four round differential trail.

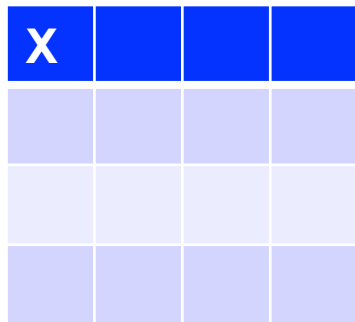
## 2.3 Design Principles of AES

- ▶ **Results**

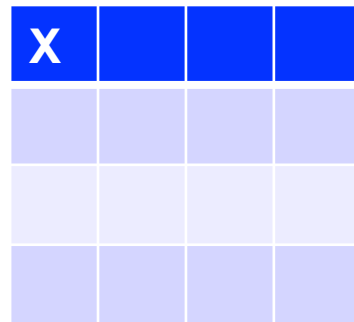
- ▶ 1. There are no 8 round correlations above  $2^{-80}$
- ▶ 2. There are no 8 round differential trails with a weight below 300
- ▶ “We consider this sufficient to resist differential and linear attacks” (designers of Rijndael)

## 2.4 AES - Square Attack – 3 Round Property

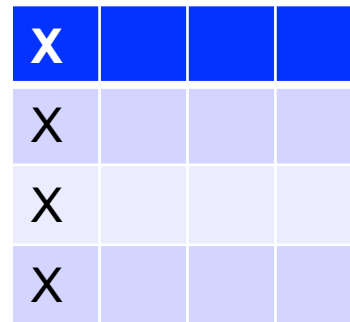
- ▶ Square Attack (aka Saturation Attack, Integral Attack, Partial Sums) – Chosen Plaintext attack originally identified by the designers of AES and improved upon since then (Ferguson et al 2000). We first describe a 3-round property. (Ref Gosh PhD Thesis)
- ▶ We input  $2^{18}$  plaintexts which are all 0 except for the first byte which varies over all  $2^8$  values. We track the XOR of the bytes at each byte position through 3 rounds of AES. The XOR of the bytes is 0 at every byte position through 3 rounds. X means byte position is active.



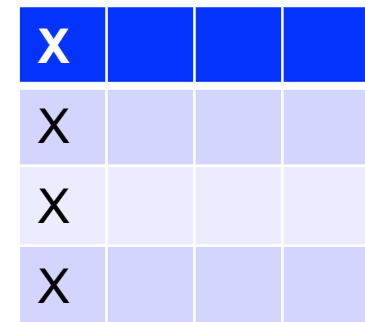
SubBytes



ShiftRows



MixColumns



AddRoundKey

Round 1  
Input  $2^{18}$  plaintexts  
First step is AddRoundKey (not shown)

## 2.5 AES - Square Attack – 3 Round Property

X			
X			
X			
X			

SubBytes

X			
			X
		X	
	X		

ShiftRows

X	X	X	X
X	X	X	X
X	X	X	X
X	X	X	X

MixColumns

X	X	X	X
X	X	X	X
X	X	X	X
X	X	X	X

AddRoundKey

Round 2  
Tracking Active Byte Positions

- Turns out the property of XOR byte positions = 0 goes through to 3 rounds by analyzing the properties of AES's third round application of the linear function MixColumns (next slide)

## 2.5 AES - Square Attack – 3 Round Property

X	X	X	X
X	X	X	X
X	X	X	X
X	X	X	X

SubBytes

$x \downarrow$ 0	X	X	X
$x \downarrow$ 1	X	X	X
$x \downarrow$ 2	X	X	X
$x \downarrow$ 3	X	X	X

ShiftRows

Round 3  
Tracking Active Byte Positions

$y \downarrow$ 0			

MixColumns


AddRoundKey

Let  $y \downarrow 0$  be the first byte

of the MixColumns operation to the first column

$$y_0^i = 02_x \cdot x_0^i \oplus 03_x \cdot x_1^i \oplus x_2^i \oplus x_3^i$$

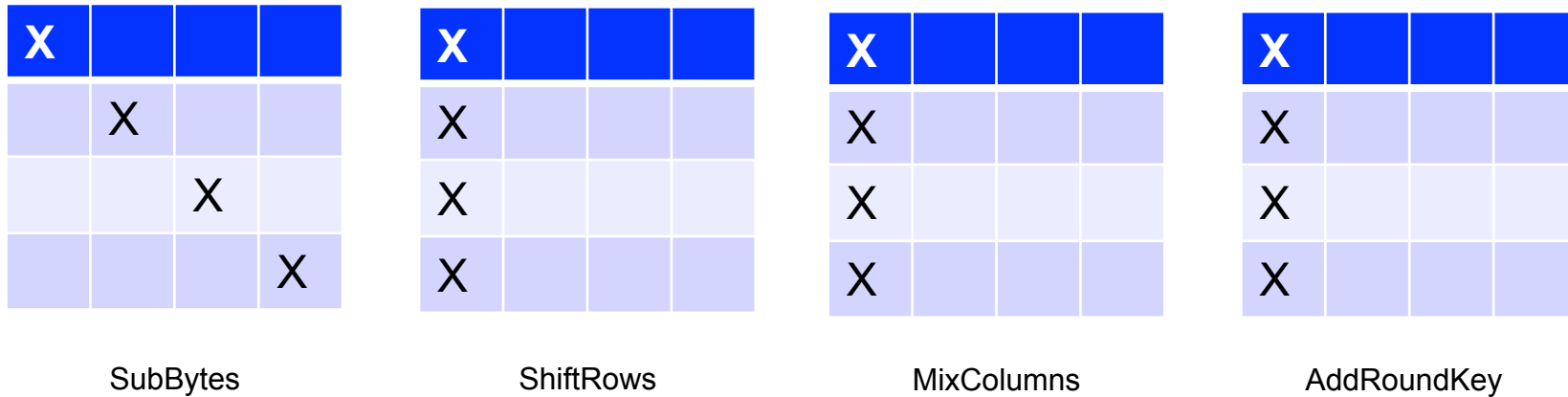
$$y_0^0 \oplus y_0^1 \oplus \dots \oplus y_0^{255} = 02_x \cdot x_0^0 \oplus 03_x \cdot x_1^0 \oplus x_2^0 \oplus x_3^0 \oplus 02_x \cdot x_0^1 \oplus 03_x \cdot x_1^1 \oplus x_2^1 \oplus x_3^1 \oplus \dots \oplus 02_x \cdot x_0^{255} \oplus 03_x \cdot x_1^{255} \oplus x_2^{255} \oplus x_3^{255}$$

$$y_0^0 \oplus y_0^1 \oplus \dots \oplus y_0^{255} = 02_x \cdot \bigoplus_{i=0}^{255} x_0^i \oplus 03_x \cdot \bigoplus_{i=0}^{255} x_1^i \oplus 01_x \cdot \bigoplus_{i=0}^{255} x_2^i \oplus 01_x \cdot \bigoplus_{i=0}^{255} x_3^i$$

$$= 02_x \cdot 00_x \oplus 03_x \cdot 00_x \oplus 00_x \oplus 00_x$$

$$= 00_x$$

## 2.6 AES - Square Attack – 4 Round Property



4-Round Property  
Add New First Round  $2^{132}$  Plaintexts at Beginning of 3-Round Property

- ▶ We can extend this to a 4-round property by using  $2^{132}$  plaintexts where the 4 active bytes vary over all possible  $2^{132}$  byte combinations
- ▶ At the end of 4-rounds the XOR of the bytes at each byte position is still 0
- ▶ We can add a key assumption at rounds 5 and 6 to get a 6 round attack – work factor  $2^{144}$  an improvement due to Ferguson et. al. 2001 from the original  $2^{172}$



## 2.7 AES - Square Attack

- ▶ Ref: “Improved “Partial Sums” – based Square Attack on AES”, Tunstall (2012)

	Rounds	Key Length	Memory	Acquisitions	Complexity
[5]	5	generic	–	$2^{11}$	$2^{40}$
This paper	5	128	–	$2^8$	$2^{38}$
This paper	5	192	–	$2 \cdot 2^8$	$2^{38.5}$
This paper	5	256	–	$2 \cdot 2^8$	$2^{39}$
[5]	6	generic	–	$5 \cdot 2^{32}$	$2^{72}$
[7]	6	generic	$2^{32}$	$6 \cdot 2^{32}$	$2^{44}$
This paper	6	128	$2^{40}$	$2^{32}$	$2^{42}$
This paper	6	192	–	$2 \cdot 2^{32}$	$2^{42.5}$
This paper	6	256	–	$2 \cdot 2^{32}$	$2^{43}$
[10]	7	192	$2^{32}$	$2^{32}$	$2^{176}$
Ferguson et al. [7]	7	192	$2^{32}$	$19 \cdot 2^{32}$	$2^{155}$
This paper	7	192	–	$2 \cdot 2^{32}$	$2^{154}$
[10]	7	256	$2^{32}$	$2^{32}$	$2^{192}$
[7]	7	256	$2^{32}$	$21 \cdot 2^{32}$	$2^{172}$
This paper	7	256	–	$2 \cdot 2^{32}$	$2^{171}$

Ref: “Implementation and Improvement of the Partial Sum Attack on 6-round AES”, Alda, Aragona, Nicolodi, Sala (2014)

Number of $\bar{\Delta}$ -sets	Average time (days)	Memory (GB)
2	12.1	1.028
3	11.5	1.542

## 2.8 Full AES - Biclique Attack

- ▶ Ref: Analysis of Block Cipher Constructions against Biclique and Multiset Attacks, PhD Thesis by Mohona Ghosh, 2016
- ▶ Summary: Key recovery with bicliques for full AES. CC=Chosen Ciphertext, CP=Chosen Plaintext

Algorithm	Rounds	Data Complexity	Time Complexity	Biclique length (rounds)	Ref.
AES-128	10	$2^{88}$ CC	$2^{125.69\dagger}$	2.5	[48]
	10	$2^{88}$ CC	$2^{126.16}$	2.5	[39]
	10	$2^{72}$ CC	$2^{126.72}$	2.5	[5]
	10	$2^4$ CP	$2^{126.89}$	2.5	[38]
AES-192	12	$2^{80}$ CC	$2^{190.16}$	3.5	[39] [5]
	12	$2^{48}$ CC	$2^{190.28}$	3.5	[5]
AES-256	14	$2^{40}$ CC	$2^{254.42}$	3.5	[39]
	14	$2^{64}$ CC	$2^{254.53}$	3.5	[5]

<sup>†</sup> Our analysis estimates the cost as  $2^{125.98}$ .

## 3.1 Quantum Cryptanalysis of AES

- ▶ Ref: “Applying Grover’s algorithm to AES: quantum resource estimates”, Grassl, Langenberg, Roetteler, Steinwandt Quant-ph 1512.04965 15 December, 2015
- ▶ Problem: Given AES-128 and 3 plaintext/ciphertext pairs, find the secret key  $K$
- ▶ Solution (roughly):
  - Step 1:  $|\psi_1\rangle = \frac{1}{\sqrt{2^{128}}} \sum_{K \in \mathcal{K}} |K\rangle |0\rangle$  uniform superposition of all  $2^{128}$  keys
  - Step 2:  $|\psi_2\rangle = \frac{1}{\sqrt{2^{128}}} \sum_{K \in \mathcal{K}} |K\rangle |AES_k(m)\rangle$  computation of AES with key  $K$  to plaintext  $m$
  - Step 3:  $|\psi_3\rangle = \frac{1}{\sqrt{2^{128}}} \sum_{K \in \mathcal{K}} |K\rangle |AES_k(m)=c\rangle$  test equality with known ciphertext  $c$
  - Step 4: apply Grover’s algorithm to find which key  $K$  gives the equality value 1 versus the inequality value 0
- ▶ Main costs: Computation of AES, Computation of Grover’s algorithm

## 3.2 Quantum Cryptanalysis of AES (Contd)

- ▶ Ref: “Applying Grover’s algorithm to AES: quantum resource estimates”, Grassl, Langenberg, Roetteler, Steinwandt Quant-ph 1512.04965 15 December, 2015
- ▶ Grover’s Cost (well known):
  - To find 1 item in an  $N$  long list costs  $\mathcal{O}(\sqrt{N})$ . Here  $N=2^{128}, 2^{192}, 2^{256}$
- ▶ AES- $k$  cost (Note quantum circuits are fully reversible)

	#gates			depth		#qubits	
	NOT	CNOT	Toffoli	$T$	overall	storage	ancillae
128	176	21,448	20,480	5,760	12,636	320	96
192	136	17,568	16,384	4,608	10,107	256	96
256	215	27,492	26,624	7,488	16,408	416	96

**Table 1.** Quantum resource estimates for the key expansion phase of AES- $k$ , where  $k \in \{128, 192, 256\}$ .

### 3.3 Quantum Cryptanalysis of AES (Contd)

- ▶ Ref: “Applying Grover’s algorithm to AES: quantum resource estimates”, Grassl, Langenberg, Roetteler, Steinwandt Quant-ph 1512.04965 15 December, 2015

	#gates		depth		#qubits
	$T$	Clifford	$T$	overall	
Initial	0	0	0	0	128
Key Gen	143,360	185,464	5,760	12,626	320
10 Rounds	917,504	1,194,956	44,928	98,173	536
Total	1,060,864	1,380,420	50,688	110,799	984

**Table 2.** Quantum resource estimates for the implementation of AES-128.

### 3.4 Quantum Cryptanalysis of AES (Contd)

- ▶ Ref: “Applying Grover’s algorithm to AES: quantum resource estimates”, Grassl, Langenberg, Roetteler, Steinwandt Quant-ph 1512.04965 15 December, 2015



	#gates		depth		#qubits
	<i>T</i>	Clifford	<i>T</i>	overall	
Initial	0	0	0	0	192
Key Gen	114,688	148,776	4,608	10,107	256
12 Rounds	1,089,536	1,418,520	39,744	86,849	664
Total	1,204,224	1,567,296	44,352	96,956	1,112

**Table 3.** Quantum resource estimates for the implementation of AES-192. The lower gate count in Key Gen and the lower depth, when compared to AES-128, arises from using the additional available space to store intermediate results and to parallelize parts of the circuit.

## 3.5 Quantum Cryptanalysis of AES (Contd)

▶ Ref: “Applying Grover’s algorithm to AES: quantum resource estimates”, Grassl, Langenberg, Roetteler, Steinwandt Quant-ph 1512.04965 15 December, 2015

▶ :

	#gates		depth		#qubits
	<i>T</i>	Clifford	<i>T</i>	overall	
Initial	0	0	0	0	256
Key Gen	186,368	240,699	7,488	16,408	416
14 Rounds	1,318,912	1,715,400	52,416	114,521	664
Total	1,505,280	1,956,099	59,904	130,929	1,336

**Table 4.** Quantum resource estimates for the implementation of AES-256.

## 3.6 Quantum Differential and Linear Cryptanalysis

- ▶ Ref: “Quantum Differential and Linear Cryptanalysis”, Kaplan, Leurent, Leverrier, Naya-Plasencia (2017)
- ▶ The authors examine differential and linear cryptanalysis in two settings defined as follows  
(PRP = Pseudo Random Permutation  
PRF = Pseudo Random Function)
- ▶ **Standard security:** a block cipher is *standard secure* against quantum adversaries if no efficient quantum algorithm can distinguish the block cipher from PRP (or a PRF) by making only *classical* queries (Q1) (i.e. can make classical encryption queries)
- ▶ **Quantum security:** a block cipher is *quantum secure* against quantum adversaries if no efficient quantum algorithm can distinguish the block cipher from PRP (or a PRF) even by making *quantum* queries (Q2) (i.e. can make quantum encryption queries)



## 3.7 Quantum Differential and Linear Cryptanalysis

- ▶ Ref: “Quantum Differential and Linear Cryptanalysis”, Kaplan, Leurent, Leverrier, Naya-Plasencia (2017)
- ▶ Results:
- ▶ Differential cryptanalysis and linear cryptanalysis usually offer a quadratic gain in the Q2 model over the classical model.
  - If a block cipher is resistant to a classical linear or differential cryptanalysis attack costing at least  $2^k$  then it is also resistant to the corresponding quantum linear or differential cryptanalysis attacks costing at least  $2^{k/2}$
- ▶ In the Q1 model cryptanalytic attacks might offer little gain over the classical model when the key-length is the same as the block length (e.g. AES-128)
- ▶ The gain of cryptanalytic attacks in the Q1 model can be quite significant (similar to the Q2 model) when the key length is longer (e.g. AES-256) than the block length

## 3.8 Breaking Symmetric Cryptosystems using Quantum Period Finding

- ▶ Ref: “Breaking Symmetric Cryptosystems using Quantum Period Finding”, Kaplan, Leurent, Leverrier, Naya-Plasencia (2016)
- ▶ Results: CBC-MAC, GMAC, GCM and PMAC, OCB are all broken by forgery attacks using their method
- ▶ The author’s take advantage of Simon’s problem and algorithm:

Simon’s problem: Given a Boolean function  $f : \{0, 1\}^n \rightarrow \{0, 1\}^n$  and the promise that there exists  $s \in \{0, 1\}^n$  such that for any  $(x, y) \in \{0, 1\}^n$ ,  $[f(x) = f(y)] \Leftrightarrow [x \oplus y \in \{0^n, s\}]$ , the goal is to find  $s$ .

- ▶ Note: Simon’s algorithm (next slide) was one of the first to show exponential speedup of a quantum algorithm.

## 3.9 Breaking Symmetric Cryptosystems using Quantum Period Finding

- ▶ Ref: “Breaking Symmetric Cryptosystems using Quantum Period Finding”, Kaplan, Leurent, Leverrier, Naya-Plasencia (2016)
- ▶ Simon’s Problem can be solved using Simon’s Algorithm which is repeated  $O(n)$  times

1. Starting with a  $2n$ -qubit state  $|0\rangle|0\rangle$ , one applies a Hadamard transform  $H^{\otimes n}$  to the first register to obtain the quantum superposition

$$\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle|0\rangle.$$

2. A quantum query to the function  $f$  maps this to the state

$$\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle|f(x)\rangle.$$

3. Measuring the second register in the computational basis yields a value  $f(z)$  and collapses the first register to the state:

$$\frac{1}{\sqrt{2}}(|z\rangle + |z \oplus s\rangle).$$

4. Applying again the Hadamard transform  $H^{\otimes n}$  to the first register gives:

$$\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{y \cdot z} (1 + (-1)^{y \cdot s}) |y\rangle.$$

5. The vectors  $y$  such that  $y \cdot s = 1$  have amplitude 0. Therefore, measuring the state in the computational basis yields a random vector  $y$  such that  $y \cdot s = 0$ .

## 3.10 Breaking Symmetric Cryptosystems using Quantum Period Finding

- ▶ Ref: “Breaking Symmetric Cryptosystems using Quantum Period Finding”, Kaplan, Leurent, Leverrier, Naya-Plasencia (2016)
- ▶ Example: CBC-MAC

$$x_0 = 0 \quad x_i = E_k(x_{i-1} \oplus m_i) \quad \text{CBC-MAC}(M) = E_{k'}(x_\ell)$$

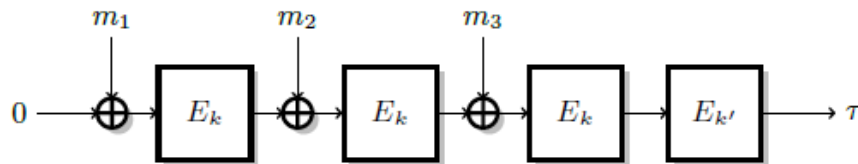


Fig. 9. Encrypt-last-block CBC-MAC.

- ▶ Fix two arbitrary message blocks  $\alpha_0, \alpha_1$ , with  $\alpha_0 \neq \alpha_1$  define the function  $f$ :

$$f : \{0, 1\} \times \{0, 1\}^n \rightarrow \{0, 1\}^n$$

$$b, x \mapsto \text{CBC-MAC}(\alpha_b \parallel x) = E_{k'}(E_k(x \oplus E_k(\alpha_b)))$$

- ▶ Note: Assumption is that we have quantum access to CBC-MAC using  $E \downarrow k$  and  $E \uparrow k$

## 3.11 Breaking Symmetric Cryptosystems using Quantum Period Finding

▶ Ref: “Breaking Symmetric Cryptosystems using Quantum Period Finding”, Kaplan, Leurent, Leverrier, Naya-Plasencia (2016)

▶ Example: CBC-MAC

▶ Simon’s algorithm returns “s” which is defined as:

$$1 \parallel E_k(\alpha_0) \oplus E_k(\alpha_1)$$

▶  $CBC-MAC(\alpha \parallel m) = E_k \left( E_k(E_k(\alpha) \oplus m) \right)$

▶ Let  $T_0 = CBC-MAC(\alpha \parallel m_0)$  for an arbitrary message block  $m_0$

▶ Let  $T_1 = CBC-MAC(\alpha \parallel m_0 \oplus E_k(\alpha) \parallel E_k(\alpha))$

▶ Then  $T_0 = T_1$  (i.e. a forgery) since we know  $E_k(\alpha) \oplus E_k(\alpha)$

## 4. References

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## 5. Questions?

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